Tabelas de Integrais Indefinidas

Observação: Em todas as fórmulas, a constante arbitrária é omitida; a, b, c, α representam números reais e m, n, p, q inteiros positivos. Quando a^2 aparece no integrando, a deve ser tomado como um número positivo, ln( ) pode sempre ser substituído por $\ln|\ |$.

1. $\int c\,dx = cx$

2. $\int cf(x)\,dx = c\int f(x)\,dx$

3. $\int \big(f(x) + g(x)\big)dx = \int f(x)\,dx + \int g(x)\,dx$

4. $\int f(x)g'(x)\,dx = f(x)g(x) - \int g(x)f'(x)\,dx$

5. $\int u\,dv = uv - \int v\,du$

6. $\int x^a\,dx = \dfrac{x^{a+1}}{a+1}, \quad a \neq 1$

7. $\int \dfrac{1}{x}\,dx = \ln|x|$

8. $\int \dfrac{f'(x)}{f(x)}\,dx = \ln|f(x)|$

9. $\int e^{ax}\,dx = \dfrac{e^{ax}}{a}$

10. $\int a^x dx = \frac{a^x}{\ln(a)}$

11. $\int \ln(x)dx = x\ln(x) - x$

12. $\int \log_a(x)dx = \frac{1}{\ln(a)}\left[x\ln(x) - x\right] = x\log_a(x) - \frac{x}{\ln(a)}; x \neq 1$

13. $\int \frac{dx}{x^2 + a^2} = \frac{1}{a} tg^{-1}\left(\frac{x}{a}\right) = -\frac{1}{a} Cotg^{-1}\left(\frac{x}{a}\right)$

14. $\int \frac{dx}{x^2 - a^2} = -\frac{1}{a} tgh^{-1}\left(\frac{x}{a}\right) = \frac{1}{2a}\ln\left|\frac{x-a}{x+a}\right|$

15. $\int \frac{dx}{a + bx^2} = \begin{cases} \frac{1}{\sqrt{ab}} tg^{-1}\left(\frac{x\sqrt{ab}}{a}\right); ab > 0 \\ \frac{1}{2\sqrt{-ab}}\ln\left(\frac{\sqrt{-bx} + \sqrt{a}}{\sqrt{-bx} - \sqrt{a}}\right); a > 0; b < 0 \end{cases}$

16. $\int \frac{xdx}{a + bx^2} = \frac{1}{2b}\ln\left(\frac{a + bx^2}{b}\right)$

17. $\int \frac{dx}{(a + bx^2)^m} = \frac{1}{2a(m-1)} \times \frac{1}{(a + bx^2)^{m-1}} + \frac{2m-3}{2(m-1)a} \times \int \frac{dx}{(a + bx^2)^{m-1}}; m > 1$

18. $\int \frac{xdx}{(a + bx^2)^m} = \frac{-1}{2b(m-1)((a + bx^2)^{m-1}}; m > 1$

19. $\int \frac{dx}{\sqrt{a^2 - x^2}} = sen^{-1}\left(\frac{x}{a}\right) = -\cos^{-1}\left(\frac{x}{a}\right)$

20. $\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln(x + \sqrt{x^2 \pm a^2})$

21. $\int \frac{dx}{x\sqrt{x^2 - a^2}} = \frac{1}{a}\cos^{-1}\left(\frac{a}{x}\right)$

22. $\int \frac{dx}{x\sqrt{a^2 \pm x^2}} = -\frac{1}{a}\ln\left(\frac{a + \sqrt{a^2 + x^2}}{x}\right)$

23. $\int \sqrt{x^2 \pm a^2}\, dx = \frac{1}{2}\left(x\sqrt{x^2 \pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2 \pm a^2}\right)\right)$

24. $\int \frac{\sqrt{x^2 + a^2}}{x} dx = \sqrt{x^2 + a^2} - a\ln\left(\frac{a + \sqrt{x^2 + a^2}}{x}\right)$

25. $\int \frac{\sqrt{x^2 - a^2}}{x} dx = \sqrt{x^2 - a^2} - a\cos^{-1}\left(\frac{a}{x}\right)$

26. $\int \frac{xdx}{\sqrt{x^2 \pm a^2}} = \sqrt{x^2 \pm a^2}$

27. $\int x\sqrt{x^2 \pm a^2}\, dx = \frac{1}{3}\left(x^2 \pm a^2\right)^{3/2}$

28. $\int \left(x^2 \pm a^2\right)^{3/2} dx = \frac{1}{8}\left[2x\left(x^2 \pm a^2\right)^{3/2} \pm 3a^2 x\sqrt{x^2 \pm a^2} + 3a^4 \ln\left(x + \sqrt{x^2 \pm a^2}\right)\right]$

29. $\int \frac{dx}{\left(x^2 \pm a^2\right)^{3/2}} = \frac{\pm x}{a^2\sqrt{x^2 \pm a^2}}$

30. $\int x^2\sqrt{(x^2 \pm a^2)}dx = \frac{x}{4}\left(x^2 \pm a^2\right)^{3/2} \mp \frac{a^2 x}{8}\sqrt{(x^2 \pm a^2)} - \frac{a^4}{8}\ln\left(x + \sqrt{(x^2 \pm a^2)}\right)$

31. $\int \sqrt{a^2 - x^2}\,dx = \frac{1}{2}\left[x\sqrt{a^2 - x^2} + a^2 sen^{-1}\left(\frac{x}{a}\right)\right]$

32. $\int \frac{\sqrt{a^2 - x^2}}{x}dx = \sqrt{a^2 - x^2} - a\ln\left(\frac{a + \sqrt{a^2 - x^2}}{x}\right)$

33. $\int \frac{1}{x\sqrt{a^2 - x^2}}dx = -\frac{1}{a}\ln\left(\frac{a + \sqrt{a^2 - x^2}}{x}\right)$

34. $\int \frac{x}{\sqrt{a^2 - x^2}}dx = -\sqrt{a^2 - x^2}$

35. $\int f(x, \sqrt{x^2 + a^2})dx = a\int f(a \times tg(u), a \times \sec(u))\sec^2(u)du\,; \quad x = a \times tg(u)$

36. $\int f(x, \sqrt{x^2 - a^2})dx = a\int f(a \times \sec(u), a \times tg(u))\sec(u) \times tg(u)du\,; \quad x = a \times \sec(u)$

37. $\int f(x, \sqrt{a^2 - x^2})dx = -a\int f(a \times \cos(u), a \times sen(u))sen(u)du\,; \quad x = a \times \cos(u)$

38. $\int f(x, X)dx = \frac{1}{a}\int f\left(\frac{y - b}{a}, \frac{y^2 + d}{a}\right)dy$

$x = (y - b)/a\,; \qquad d = ac - b^2\,; \qquad X = ax^2 + 2bx + c$

39. $\int x^n (a + bx)^n\,dx = \frac{x^{n+1}(a + bx)^n}{m + n + 1} + \frac{an}{m + n + 1}\int x^n (a + bx)^{n-1}\,dx$

40. $\int \frac{dx}{a+bx} = \frac{1}{b}\ln(a+bx)$

41. $\int \frac{xdx}{a+bx} = \frac{1}{b^2}\left[a+bx-a\ln(a+bx)\right]$

42. $\int \frac{xdx}{(a+bx)^2} = \frac{1}{b^2}\left[\ln(a+bx)+\frac{a}{a+bx}\right]$

43. $\int \frac{xdx}{(a+bx)^m} = \frac{1}{b^2}\left[\frac{-1}{(m-2)(a+bx)^{m-2}}+\frac{a}{(m-1)(a+bx)^{m-1}}\right]; \quad m \geq 3$

44. $\int \sqrt{a+bx}dx = \frac{2}{3b}\sqrt{(a+bx)^2}$

45. $\int x^m\sqrt{a+bx}dx = \frac{2}{(2m+3)b}\left[x^m\sqrt{(a+bx)^2}-ma\int x^{m-1}\sqrt{a+bx}dx\right]$

46. $\int \frac{dx}{x^m\sqrt{(a+bx)}} = \frac{-\sqrt{(a+bx)}}{a(m-1)x^{m-1}}-\frac{(2m-3)b}{(2m-2)a}\int \frac{dx}{x^{m-1}\sqrt{(a+bx)}}; \quad m \neq 1$

47. $\int f(x,\sqrt{a+bx})dx = \frac{2}{b}\int f\left(\frac{z^2-a}{b},z\right)zdz; \quad z^2 = a+bx$

48. $\int \frac{dx}{a^2+x^2} = \frac{1}{3a^2}\left[\frac{1}{2}\ln\left[\frac{(a+x)^2}{a^2-ax+x^2}\right]+\sqrt{3}\ tg^{-1}\left(\frac{2x-a}{a\sqrt{3}}\right)\right]$

49. $\int sen(x)dx = -\cos(x)$

50. $\int \cos(x)dx = sen(x)$

51. $\int tg(x)dx = -\ln(\cos(x))$

52. $\int \cot g(x)dx = \ln(sen(x))$

53. $\int \sec(x)dx = \ln\left[tg\left(\frac{x}{2}+\frac{\pi}{2}\right)\right]$

54. $\int \cos ec(x)dx = \ln\left[tg\left(\frac{x}{2}\right)\right]$

55. $\int sen^2(x)dx = \frac{1}{2}\left[x-\cos(x)sen(x)\right]$

56. $\int sen^m(x)dx = \frac{-\cos(x)]sen(x)^{m-1}}{m}+\frac{m-1}{m}\int sen(x)^{m-2}dx$

57. $\int \cos^2(x)dx = \frac{1}{2}x+\frac{1}{4}sen(2x)$

58. $\int \cos^m(x)dx = \frac{sen(x)\cos^{m-1}(x)}{m}+\frac{m-1}{m}\int \cos^{m-2}(x)dx$

59. $\int \frac{dx}{\cos^2(x)} = \int \sec^2(x)dx = tg(x)$

60. $\int \frac{dx}{\cos^m(x)} = \frac{sen(x)}{(m-1)\cos^{m-1}(x)}+\frac{m-2}{m-1}\int \frac{dx}{\cos^{m-2}(x)}; \quad m>1$

61. $\int \frac{dx}{sen^2(x)} = \int \cos ec^2(x)dx = -\cot g(x)$

62. $\int \frac{dx}{sen^m(x)} = \frac{-\cos(x)}{(m-1)sen^{m-1}(x)}+\frac{m-2}{m-1}\int \frac{dx}{sen^{m-2}(x)}; \quad m>1$

63. $\int \frac{dx}{1\pm sen(x)} = \mp tg\left(\frac{\pi}{4}\mp\frac{x}{2}\right)$

64. $\int \frac{dx}{1+\cos(x)} = tg\left(\frac{x}{2}\right)$

65. $\int \frac{dx}{1-\cos(x)} = -\cot g\left(\frac{x}{2}\right)$

66. $\int \frac{dx}{a+b\ sen(x)} = \begin{cases} \frac{1}{\sqrt{b^2-a^2}} \ln\left(\frac{a\ tg\left(\frac{x}{2}\right)+b-\sqrt{b^2-a^2}}{a\ tg\left(\frac{x}{2}\right)+b+\sqrt{b^2-a^2}}\right); b^2 > a^2 \\ \frac{2}{\sqrt{a^2-b^2}} tg^{-1}\left(\frac{a\ tg\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right); a^2 > b^2 \end{cases}$

67. $\int \frac{dx}{a+b\ \cos(x)} = \begin{cases} \frac{1}{\sqrt{b^2-a^2}} \ln\left(\frac{\sqrt{b^2-a^2}\ tg\left(\frac{x}{2}\right)+a+b}{\sqrt{b^2-a^2}\ tg\left(\frac{x}{2}\right)-a-b}\right); b^2 > a^2 \\ \frac{2}{\sqrt{a^2-b^2}} tg^{-1}\left(\frac{\sqrt{a^2-b^2}\ tg\left(\frac{x}{2}\right)}{a+b}\right); a^2 > b^2 \end{cases}$

68. $\int sen(nx) \times sen(mx)dx = \frac{sen(m-n)x}{2(m-n)} - \frac{sen(m+n)x}{2(m+n)}; m^2 \neq n^2$

69. $\int sen(nx) \times \cos(mx)dx = \frac{\cos(m-n)x}{2(m-n)} - \frac{\cos(m+n)x}{2(m+n)}; m^2 \neq n^2$

70. $\int \cos(nx) \times \cos(mx)dx = \frac{sen(m-n)x}{2(m-n)} + \frac{sen(m+n)x}{2(m+n)}; m^2 \neq n^2$

71. $\int tg^{n}(x)dx = \frac{tg^{n-1}(x)}{n-1} - \int tg^{n-2}(x)dx; n \neq 1$

72. $\int \frac{dx}{sen(x)\cos(x)} = \ln(tg(x))$

73. $\int \frac{dx}{sen(x)\cos^{m}(x)} = \frac{1}{(m-1)\cos^{m-1}(x)} + \int \frac{dx}{sen(x)\cos^{m-1}(x)}; m > 1$

74. $\int x^{m} sen(x)dx = -x^{m}\cos(x) + m\int x^{m-1}\cos(x)dx$

75. $\int x^{m}\cos(x)dx = x^{m} sen(x) - m\int x^{m-1} sen(x)dx$

76. $\int sen^{-1}(x)dx = x\ sen^{-1}(x) + \sqrt{1-x^{2}}$

77. $\int \cos^{-1}(x)dx = x\ \cos^{-1}(x) - \sqrt{1-x^{2}}$

78. $\int tg^{-1}(x)dx = x\ tg^{-1}(x) - \frac{1}{2}\ln\left(1+x^{2}\right)$

79. $\int \cot g^{-1}(x)dx = x\ \cot g^{-1}(x) + \frac{1}{2}\ln\left(1+x^{2}\right)$

80. $\int \left(sen^{-1}(x)\right)^{2}dx = x\ \left(sen^{-1}(x)\right)^{2} - 2x + 2\sqrt{1-x^{2}}\ sen^{-1}(x)$

81. $\int \left(\cos^{-1}(x)\right)^{2}dx = x\ \left(\cos^{-1}(x)\right)^{2} - 2x - 2\sqrt{1-x^{2}}\ \cos^{-1}(x)$

82. $\int x^{n} sen^{-1}(x)dx = \frac{x^{n+1} sen^{-1}(x)}{n+1} - \frac{1}{n+1}\int \frac{x^{n+1}}{\sqrt{1-x^{2}}}dx$

83. $\int x^n \cos^{-1}(x)dx = \frac{x^{n+1}\cos^{-1}(x)}{n+1} + \frac{1}{n+1}\int \frac{x^{n+1}}{\sqrt{1-x^2}}dx$

84. $\int x\ln(x)dx = \frac{x^2}{2}\ln(x) - \frac{x^2}{4}$

85. $\int x^m \ln(x)dx = \frac{x^{Mn+1}}{m+1} - \frac{x^{m+12}}{(m+1)^2}; m \neq 1$

86. $\int (\ln(x))^q dx = x(\ln(x))^q - q\int (\ln(x))^{q-1} dx$

87. $\int \frac{(\ln(x))^q}{x} dx = \frac{(\ln(x))^{q+1}}{q+1}$

88. $\int \frac{dx}{x\ln(x)} dx = \ln(\ln(x))$

89. $\int x^m (\ln(x))^q dx = \frac{x^{m+1}(\ln(x))^q}{m+1} - \frac{q}{m+1}\int x^m (\ln(x))q^{-1}dx; m \neq 1$

90. $\int sen(\ln(x))dx = \frac{1}{2}x\ sen(\ln(x)) - \frac{1}{2}x\ \cos(\ln(x))$

91. $\int \cos(\ln(x))dx = \frac{1}{2}x\ sen(\ln(x)) + \frac{1}{2}x\ \cos(\ln(x))$

92. $\int x\ e^{ax}dx = \frac{e^{ax}}{a^2}(ax-1)$

93. $\int x^m e^{ax}dx = \frac{x^m e^{ax}}{a} - \int x^{m-1}e^{ax}dx;\ \ m > 0$

94. $\int \frac{e^{ax}}{x^m} dx = -\frac{e^{ax}}{(m-1)x^{m-1}} + \frac{a}{m-1}\int \frac{e^{ax}}{x^{m-1}} dx; m > 1$

95. $\int e^{ax} \ln(x) dx = \frac{e^{ax} \ln(x)}{a} - \frac{1}{a}\int \frac{e^{ax}}{x} dx$

96. $\int e^{ax} sen(nx) dx = \frac{e^{ax}(a\ \ sen(nx) - n\cos(nx))}{a^2 + n^2}$

97. $\int e^{ax} \cos(nx) dx = \frac{e^{ax}(a\ \ \cos(nx) + n\ \ sen(nx))}{x^2 + n^2}$

98. $\int \frac{dx}{a + be^{ax}} = \frac{x}{a} - \frac{1}{aq} \ln(a + be^{ax})$

99. $\int senh(x) dx = \cosh(x)$

100. $\int \cosh(x) dx = senh(x)$

101. $\int tgh(x) dx = \ln \cosh(x)$

102. $\int \cot gh(x) dx = \ln\ \ senh(x)$

103. $\int \sec h(x) dx = 2tg^{-1}e^x = tg^{-1}(senh(x))$

104. $\int \cos ech(x) dx = \ln\left(tgh\left(\frac{x}{2}\right)\right)$

105. $$\int f(sen(x))dx = \begin{cases} 2\int f\left(\dfrac{2z}{1+z^2}\right)\dfrac{dz}{1+z^2}; z = tg\left(\dfrac{x}{2}\right) \\ \int f(u)\dfrac{du}{\sqrt{1-u^2}}; u = sen(x) \end{cases}$$

106. $$\int f(\cos(x))dx = \begin{cases} 2\int f\left(\dfrac{1-z^2}{1+z^2}\right)\dfrac{dz}{1+z^2}; z = tg\left(\dfrac{x}{2}\right) \\ -\int f(u)\dfrac{du}{\sqrt{1-u^2}}; u = \cos(x) \end{cases}$$

107. $$\int f(sen(x),\cos(x))dx = \begin{cases} 2\int f\left(\dfrac{2z}{1+z^2},\dfrac{1-z^2}{1+z^2}\right)\dfrac{dz}{1+z^2}; z = tg\left(\dfrac{x}{2}\right) \\ \int f\left(u,\sqrt{1-u^2}\right)\dfrac{du}{\sqrt{1-u^2}}; u = sen(x) \end{cases}$$

108. $$\int_0^{\pi/2} sen^n(x)dx = \int_0^{\pi/2} \cos^n(x)dx = \begin{cases} \dfrac{1}{2}\times\dfrac{3}{4}\times\cdots\times\dfrac{n-1}{n}\times\dfrac{\pi}{2}; n = 2;4;6;\ldots \\ \dfrac{2}{3}\times\dfrac{4}{5}\times\cdots\times\dfrac{n-1}{n}; n = 3;5;7;\ldots \end{cases}$$

109. $$\int_0^{\pi/2} sen^n(x)\cos^2(x)dx = \begin{cases} \dfrac{1\times3\times\cdots\times(m-1)\times1\times3\cdots\times(n-1)}{2\times4\times\cdots\times(m+n)}\times\dfrac{\pi}{2}; m,n = 2;4;\ldots \\ \dfrac{2\times4\times\cdots\times(m-1)}{(n+1)\times(n+3)\times\cdots\times(n+m)}; m = 3;5;7;\cdots, n = 1;2;3;\cdots \\ \dfrac{2\times4\times\cdots\times(n-1)}{(m+1)\times(m+3)\times\cdots\times(m+n)}; m = 1;2;3;\cdots, n = 3;5;7;\cdots \end{cases}$$

110. $$\int_{-\infty}^{+\infty} e^{-x^2/2}dx = \sqrt{2\pi}$$